The Lifetime Asymmetry of Polarized Muons in Flight

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Based on the parity violation in Standard Model, we study the dependence of the lifetime on the polarization of an initial-state fermion in weak interactions. The fermion lifetime was usually calculated in terms of spin states. However, after comparing spin states with helicity states and chirality states, it is pointed out that a spin state is helicity degenerate, and the spin state and the helicity state are entirely different. Using helicity states, we calculate the lifetime of polarized muons. The result shows that the lifetime of right-handed polarized muons is always greater than that of left-handed polarized muons with the same speed in flight.

PACS numbers: 12.15.-y; 11.30.Rd; 11.30.Er; 11.55.-m; 13.35.Bv;

I. INTRODUCTION

In virtue of parity violation the experiments have shown that all fermions emitted from decay processes are longitudinally polarized. It is well known that neutrinos are left-handed (LH) polarized while antineutrinos are right-handed (RH) polarized. In beta or muon decays it is found that electrons are LH polarized while positrons are RH polarized.[1,2] Based on these experimental facts, naturally, it is thought that not only the final-state fermions, but also the initial-state fermion should reveal the feature of longitudinal polarization in weak interactions. So we could further consider that the lifetime of fermions in the LH helicity state and that in the RH helicity state should be different. Namely, the lifetime asymmetry should exist in left-right handed polarized fermions. However, the experimental and theoretical study on the polarization or helicity of the initial fermions in decays has not yet been discussed fully in the literature. In a previous paper[3] we have given a first discussion of the problem on lifetime asymmetry. In this paper we continue the study of this issue. The differences among spin states, helicity states and chirality states are emphasized and the lifetime asymmetry of polarized muons will be calculated concretely.

II. THE CHARGED WEAK CURRENTS IN THE STANDARD MODEL

In the standard model (SM), all of fundamental fermions are divided into two classes, LH chirality state and RH chirality state, in order to describe the parity violation in weak interactions. The LH chirality state is different from the RH chirality state. The former is a SU(2)-doublet state whereas the latter a SU(2)-singlet state, hence they have different gauge transformations. Especially, RH chirality states have zero weak isospin and are only present in neutral weak currents. Therefore, only LH chirality states exist in charged weak currents. The interaction Lagrangians for charged weak lepton current and charged weak quark current read, respectively

$$\mathcal{L}_{\ell W} = \frac{1}{\sqrt{2}}\, g_2\, \overline{e}_L \gamma_\mu W_\mu^+ \nu_L + \frac{1}{\sqrt{2}}\, g_2\, \overline{\nu}_L \gamma_\mu W_\mu^- e_L\,, \quad (1)$$

$$\mathcal{L}_{qW} = \frac{1}{\sqrt{2}}\, g_2\, \overline{d}_L \gamma_\mu W_\mu^+ u_L + \frac{1}{\sqrt{2}}\, g_2\, \overline{u}_L \gamma_\mu W_\mu^- d_L\,, \quad (2)$$

where g_2 is the coupling constant corresponding to SU(2) and the subscript L denotes the LH chirality state. Obviously, all fermions are in the LH chirality states while all antifermions are in the RH chirality states.

For example, the weak interaction in muon decay is successfully described by four-fermion interaction Hamiltonian. We denote the matrix element by

$$M \sim \sum g^{\gamma}_{\varepsilon\mu} \langle \overline{e}_\varepsilon | \Gamma_\gamma | (\nu_e)_n \rangle \langle (\overline{\nu}_\mu)_m | \Gamma_\gamma | \mu_\mu \rangle, \quad (3)$$

where $\gamma = S, V, T$ indicates a scalar, vector or tensor interaction, and $\varepsilon, \mu = R, L$ indicate a right- or left-handed chirality of the electron or muon. The chiralities n and m of the ν_e and $\overline{\nu}_\mu$ are then determined by the values of γ, ε and μ. All coupling constants have been obtained entirely from experiments without any model assumption.[2,4] The experiments on muon decay show g_{RL}, g_{RR}, g_{LR} to be zero, and at least one of the two coupling, g^V_{LL} or g^S_{LL}, to be nonzero. The experiments on inverse muon decay provide a lower limit for pure $V - A$ interaction with $|g^V_{LL}| > 0.960$. Thus the measurements give a strong support to the SM which sets $g^V_{LL} = 1$ while all others being zero, and then indicate that the charged weak current is dominated by a coupling to left-handed chirality fermions. Therefore, the negative muon decay can be written as

$$\mu^-_L \longrightarrow e^-_L + \overline{\nu}_{eR} + \nu_{\mu_L}. \quad (4)$$

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And the matrix element (3) has the form

$$M \sim (\overline{\nu}_{\mu_L} \gamma_\mu \, \mu_{_L})(\overline{e}_{_L} \gamma_\mu \, \nu_{e_L}). \tag{5}$$

III. SPIN STATES, HELICITY STATES AND CHIRALITY STATES

There exist three kinds of spinor wave functions, i.e., the spin states, the helicity states and the chirality states. We will discuss the difference and the relation among them.

A. The spin states

The spin states are the plane wave solutions of Dirac equation, and in momentum representation for a given four-momentum p and mass m, the positive energy solution and the so-called negative energy solution are respectively

$$u_s(p) = \sqrt{\frac{E_p + m}{2E_p}} \begin{pmatrix} \varphi_s \\ \dfrac{\boldsymbol{\sigma} \cdot \boldsymbol{p}}{E_p + m} \varphi_s \end{pmatrix}, \tag{6}$$

$$v_s(p) = \sqrt{\frac{E_p + m}{2E_p}} \begin{pmatrix} \dfrac{\boldsymbol{\sigma} \cdot \boldsymbol{p}}{E_p + m} \varphi_s \\ \varphi_s \end{pmatrix}, \tag{7}$$

where $E_p > 0$, $s = 1, 2$ and φ_s are Pauli spin wave functions. The state with $s = 1$ is spin up ($\sigma_z = 1$) while the state with $s = 2$ is spin down ($\sigma_z = -1$). They are eigenstates of operator, $\frac{\omega(\boldsymbol{p}) \cdot e}{m}$, with eigenvalues ± 1, namely

$$\frac{\omega(p) \cdot e}{m} \, u_s(p) = \begin{cases} u_s(p), & (s = 1) \\ -u_s(p). & (s = 2) \end{cases} \tag{8}$$

Here $\omega(p)$ is the Pauli-Lubanski covariant spin vector and e is the four-polarization vector in the form

$$e_\alpha = \begin{cases} \boldsymbol{e}^0 + \dfrac{\boldsymbol{p} \, (\boldsymbol{p} \cdot \boldsymbol{e}^0)}{m(E_p + m)}, & (\alpha = 1, 2, 3) \\ i \, \dfrac{\boldsymbol{p} \cdot \boldsymbol{e}^0}{m}, & (\alpha = 4) \end{cases} \tag{9}$$

which is normalized ($e^2 = 1$), orthogonal to p ($e \cdot p = 0$). In the rest frame e reduces to $e^0 = (\boldsymbol{e}^0, 0) = (0, 0, 1, 0)$. In application it is frequently necessary to evaluate spin sums in the form

$$\begin{aligned} P_1(p) &= \rho_+ \, \Lambda_+(p), \quad P_2(p) = \rho_- \, \Lambda_+(p), \\ P_3(p) &= \rho_+ \, \Lambda_-(p), \quad P_4(p) = \rho_- \, \Lambda_-(p). \end{aligned} \tag{10}$$

The $\Lambda_+(p)$ and $\Lambda_-(p)$ are the positive energy projection operator and the negative energy projection operator,

$$\Lambda_+(p) = \frac{-i \, \gamma \cdot p + m}{2E_p}, \quad \Lambda_-(p) = \frac{i \, \gamma \cdot p + m}{2E_p}, \tag{11}$$

respectively and $\rho_\pm$ are spin projection operators:

$$\rho_\pm = \frac{1}{2}(1 \pm i \, \gamma_5 \, \gamma \cdot e). \tag{12}$$

The plus sign refers to $s = 1$ and the minus sign to $s = 2$. One sees that the operator $P_1(p)$ project out the positive energy states with spin up and $P_2(p)$ the positive energy states with spin down, whereas the operator $P_3(p)$ the negative energy states with spin down and $P_4(p)$ the negative energy states with spin up in its rest frame.

B. The helicity states

A helicity state is the eigenstate of the helicity of fermions and satisfies the ordinary Dirac equation[5]. If spinor φ is taken as the eigenstate of the spin component along the direction of its motion,

$$\frac{\boldsymbol{\sigma} \cdot \boldsymbol{p}}{|\boldsymbol{p}|} \, \varphi_{_h} = h \, \varphi_{_h}, \quad h = \pm 1 \tag{13}$$

then the helicity states read

$$u_{_h}(p) = \sqrt{\frac{E_p + m}{2E_p}} \begin{pmatrix} \varphi_{_h} \\ \dfrac{h|\boldsymbol{p}|}{E_p + m} \varphi_{_h} \end{pmatrix}, \tag{14}$$

and

$$\varphi_{_{+1}} = \begin{pmatrix} \cos\frac{\theta}{2} \, e^{-i\frac{\phi}{2}} \\ \sin\frac{\theta}{2} \, e^{i\frac{\phi}{2}} \end{pmatrix}, \; \varphi_{_{-1}} = \begin{pmatrix} -\sin\frac{\theta}{2} \, e^{-i\frac{\phi}{2}} \\ +\cos\frac{\theta}{2} \, e^{i\frac{\phi}{2}} \end{pmatrix}, \tag{15}$$

where θ and ϕ are the polar angles of momentum $\boldsymbol{p}$ in polar-coordinates. The state with $h = +1$ is the RH helicity state while the state with $h = -1$ is the LH helicity state. Because a helicity state is also a plane wave solutions of Dirac equation, the energy projection operator of helicity state is equal to that of spin state, namely

$$\sum_h u_h(p) \overline{u}_h(p) = \sum_{s=1}^{2} u_s(p) \overline{u}_s(p). \tag{16}$$

The projection operators of the helicity states are[6]

$$\rho_{_h} = \frac{1}{2} \left(1 \pm \frac{\boldsymbol{\Sigma} \cdot \boldsymbol{p}}{|\boldsymbol{p}|} \right). \tag{17}$$

We can see that the helicity state is entirely different from the spin state. Both the helicity eigenvalue and the projection operator of helicity state are not Lorentz invariant, and the latter is essentially a two-component operator. From Eqs. (6), (7) and (8) we can find out that a spin state with the same s but different values of h is helicity degenerate. So the spin states can not uniquely describe the helicity of fermions. On the other hand, taking the simplest case of $\boldsymbol{p} : \boldsymbol{p} = p_z$, which does not

lose the universality of problem, we have the LH helicity state $u_{_{Lh}}$ and the RH helicity state $u_{_{Rh}}$, respectively

$$u_{_{Lh}}(p) = \sqrt{\frac{E_p+m}{2E_p}}\begin{pmatrix}\varphi_2\\ \dfrac{-|\boldsymbol{p}|\,\varphi_2}{E_p+m}\end{pmatrix}, \tag{18}$$

$$u_{_{Rh}}(p) = \sqrt{\frac{E_p+m}{2E_p}}\begin{pmatrix}\varphi_1\\ \dfrac{|\boldsymbol{p}|\,\varphi_1}{E_p+m}\end{pmatrix}. \tag{19}$$

Even so comparing Eq. (12) with Eq. (17) one can also see that the projection operator of spin state is different from that of helicity state though the spin and helicity state are formally identical when $\boldsymbol{p} = p_z$.

Furthermore, we should point out emphatically that the helicity and the degree of polarization are two different concepts. The helicity h describes the mutual relation between the spin direction of fermions and its momentum direction. Because helicity changes its sign under the space inversion, it is called a pseudoscalar. The degree of polarization is the length of polarization vector $\boldsymbol{P}$ which is the ensemble average of spin vector of fermion beam. The polarization vector does not change its sign under the space inversion and so is called a pseudovector or an axial vector, like angular momentum. The helicity can only take the value of either 1 or -1 and has no meaning in its rest frame. But the degree of polarization can take any value between 0 and 1 and the value remains constant in different frame. In the spin states the polarization of fermions is described by polarization vector. It is the discrepancy between the helicity and the polarization vector that results in the spin state being different from the helicity state.

The four-polarization vector e is the relativistic generalization of three-polarization vector $\boldsymbol{P}$. It is able to prove from Eq. (12) that the vector $\boldsymbol{e}$ is the ensemble average of spin vector $\boldsymbol{\sigma} = -i\gamma_4\gamma_\mu\gamma_5$, i.e.,

$$\boldsymbol{e} = \langle\boldsymbol{\sigma}\rangle = \boldsymbol{P}. \tag{20}$$

On the other hand, however, we can see out from Eq. (9) that the property of vector $\boldsymbol{e}$ is different from that of pseudovector $\boldsymbol{P}$. For example, when $\boldsymbol{p} = p_z$ vector $\boldsymbol{e}$ reads

$$\boldsymbol{e} = \frac{E}{m}\boldsymbol{e}^0. \tag{21}$$

Differing from vector $\boldsymbol{P}$, obviously, the direction of vector $\boldsymbol{e}$ is always pointing to z axis and its value can be greater than one, i.e., $|\boldsymbol{e}| \geq 1$. Strictly speaking, only in the rest frame can the four-polarization vector e be most unambiguously defined[7]. And the spin projection operators $\rho_\pm$, which are Lorentz invariant, can only project out the states which in its rest frame have spin $s = 1$ and 2, respectively. Therefore, the Pauli-Lubanski covariant spin vector $\omega(p)$ and the four-polarization vector e, not like four-momenta, have no intuitively and definitely physical significance in the motion frame. We reach a conclusion that the polarization of fermions must be described by the helicity states which are closely related to directly observable quantity experimentally.

C. The chirality states

The chirality states are the eigenstates of chirality operator γ_5. The LH chirality state and the RH chirality state are defined as, respectively

$$u_{_{LS}}(p) = \frac{1}{2}(1+\gamma_5)u_s(p), \quad u_{_{RS}}(p) = \frac{1}{2}(1-\gamma_5)u_s(p). \tag{22}$$

In general, chirality states are different from helicity states. Only if $m = 0$ (for example neutrinos) or $E \gg m$ (in the ultrarelativistic limit) the fermions satisfy Weyl equation[3,8], the spinor φ_s must then be taken to be eigenstates of helicity operator h and the polarization is always in the direction of motion[9]. In other words, for $m = 0$ the helicity states, the chirality states and spin states are identical, i.e.

$$u^W_{Lh}(p) = u^W_{L2}(p) = u^W_2(p) = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_2\\ -\varphi_2\end{pmatrix}, \tag{23}$$

$$u^W_{Rh}(p) = u^W_{R1}(p) = u^W_1(p) = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_1\\ \varphi_1\end{pmatrix}. \tag{24}$$

The superscript W refers to it being a solution of Weyl equation.

The helicity states $u_{_{Lh}}$ and $u_{_{Rh}}$ in Eqs. (18) and (19) can be expanded as linear combination of chirality states, respectively[10]

$$\begin{aligned} u_{_{Lh}}(p) &= \frac{1}{2}(1+\gamma_5)\,u_{_{Lh}}(p) + \frac{1}{2}(1-\gamma_5)\,u_{_{Lh}}(p) \\ &= C_{LL}\,u^0_{_{L2}} + C_{LR}\,u^0_{_{R2}}, \end{aligned} \tag{25}$$

$$u_{_{Rh}}(p) = C_{RL}\,u^0_{_{L1}} + C_{RR}\,u^0_{_{R1}}, \tag{26}$$

where $u^0_{_{LS}}$ and $u^0_{_{RS}}$ are chirality states in the rest frame,

$$u^0_{_{LS}} = \frac{1}{2}\begin{pmatrix}\varphi_s\\ -\varphi_s\end{pmatrix}, \quad u^0_{_{RS}} = \frac{1}{2}\begin{pmatrix}\varphi_s\\ \varphi_s\end{pmatrix}. \tag{27}$$

The coefficients C_{LL}, C_{LR}, C_{RL} and C_{RR} as given by

$$\begin{aligned} C_{LL} = C_{RR} &= \frac{1}{\sqrt{2E_p}}(\sqrt{E_p+m}+\sqrt{E_p-m}) \\ &= \sqrt{1+\beta}, \end{aligned} \tag{28}$$

$$\begin{aligned} C_{RL} = C_{LR} &= \frac{1}{\sqrt{2E_p}}(\sqrt{E_p+m}-\sqrt{E_p-m}) \\ &= \sqrt{1-\beta}, \end{aligned} \tag{29}$$

where β is the velocity of the muons. It is obvious from Eqs. (25) and (26) that in a LH helicity state $u_{_{Lh}}(p)$ the

coefficient C_{LL} is the amplitude of LH chirality state u^0_{L2} and the C_{LR} that of RH chirality state u^0_{R2}; while in a RH helicity state $u_{Rh}(p)$ the C_{RL} that of LH chirality state u^0_{L1} and the C_{RR} that of RH chirality state u^0_{R1} in its rest frame. For LH helicity state the hidden amplitude of RH chirality state decreases with the increase of β until $C_{LR} \to 0$ when $\beta \to 1$, showing that a high-energy fermion can be LH polarized without hidden RH spinning instability. For RH helicity state the hidden amplitude of LH chirality state decreases with the increase of β. When $\beta = 0$, the hidden amplitude of LH chirality state is equal to that of RH chirality state and one even can not discriminate a rest fermion being either LH or RH polarized[11].

IV. THE LIFETIME OF POLARIZED MUONS

Now let us consider a μ^- decay process (4). The lowest order decay rate or lifetime τ for muon decays, based on the perturbation theory of weak interactions, is given by

$$\tau^{-1} = \frac{1}{(2\pi)^5}\int d^3q\; d^3k\; d^3k'\; \delta^4(p-q-k-k')\; M^2. \quad (30)$$

A. The lifetime of unpolarized muons

If the muons are unpolarized and if we do not observe the polarization of final-state fermions, then the transition matrix element, Eq. (5), is given by averaging over the muon spin and summing over all final fermion spins:

$$\begin{aligned} M^2 = \;& \frac{G^2}{2}\,\frac{1}{2}\sum_{s,s',r,r'=1}^{2}\left[\,\overline{u}_{s'}(q)\;\gamma_\lambda\;(1+\gamma_5)\;v_{r'}(k')\,\right]^2 \\ & \times\left[\,\overline{u}_r(k)\;\gamma_\lambda\;(1+\gamma_5)\;u_s(p)\,\right]^2. \end{aligned} \quad (31)$$

where p, q, k and k' are 4-momenta, while s, s', r and r' are spin indices for μ, e, ν_μ and $\bar{\nu}_e$, respectively. For the convenience of discussion below, in Eq. (31) we set

$$I = \frac{1}{2}\sum_{s=1}^{2}\sum_{r=1}^{2}\left[\,\overline{u}_r(k)\;\gamma_\lambda\;(1+\gamma_5)\;u_s(p)\,\right]^2, \quad (32)$$

which is related to the muons. By means of Eqs. (10) and (12), the evaluations of spin sums are reduced to the calculation of projection operators:

$$\sum_{s=1}^{2} u_s(p)\overline{u}_s(p) \;=\; \sum_{s=1}^{2} P_s(p) = \Lambda_+(p), \quad (33)$$

$$\sum_{s=1}^{2} v_s(p)\overline{v}_s(p) \;=\; -\sum_{s=1}^{2} P_s(p) = -\Lambda_-(p). \quad (34)$$

One sees that the explicit evaluation of spin projection operators disappear. Applying Eqs. (33), (34) and (11) as well as the trace theorems we obtain

$$M^2 = \frac{4\;G^2\;(p\cdot k')\;(q\cdot k)}{E_pE_qE_kE_{k'}}. \quad (35)$$

Substituting Eq. (35) into Eq. (30), one has

$$\tau^{-1} = \frac{4\;G^2}{(2\pi)^5}\frac{1}{E_p}\int\frac{d^3q}{E_q}\frac{d^3k}{E_k}\frac{d^3k'}{E_{k'}}\delta^4(p-q-k-k')\mathcal{F}, \quad (36)$$

where

$$\mathcal{F} = (p\cdot k')\;(q\cdot k). \quad (37)$$

Obviously decay amplitude $\mathcal{F}$ is a Lorentz-invariant matrix element. Therefore we have

$$\mathcal{F} = \mathcal{F}^0 = (p^0\cdot k')(q\cdot k), \quad p^0 = (0,0,0,im_\mu) \quad (38)$$

where $\mathcal{F}^0$ is $\mathcal{F}$ in the muon rest frame.

It is easy to see from Eq. (36) that the integration to the right of E_p^{-1} is Lorentz invariant. Neglecting electron mass, the muon lifetime τ_0 in its rest frame is given by

$$\tau_0^{-1} = \frac{G^2m_\mu^5}{192\;\pi^3}, \quad (39)$$

where m_μ is muon mass. In an arbitrary frame the muon lifetime is given by

$$\tau = \frac{\tau_0}{\sqrt{1-\beta^2}}. \quad (40)$$

We see that the lifetime is proportional to the velocity β of muons as required by special relativity[12]. So that the lifetime is not a Lorentz scalar.

B. The lifetime of polarized muons expressed by the spin states

For polarized muons the muon spin should not be averaged. In most literatures and textbooks[2,13] the polarized states of fermions were usually expressed by the spin states (6) and (7). Instead of Eqs. (33) and (34), we have

$$I_s = \sum_{r=1}^{2}\left[\overline{u}_r(k)\gamma_\lambda(1+\gamma_5)u_s(p)\right]^2, \quad (41)$$

and

$$u_s(p)\overline{u}_s(p) = P_s(p) = \frac{1}{2}(1\pm i\gamma_5\gamma\cdot e)\frac{(-i\gamma\cdot p+m_\mu)}{2E_p}, \quad (42)$$

respectively. Substituting Eq. (42) into (41) and Eq. (41) into (32) we obtain

$$M_s^2 = \frac{4\;G^2\mathcal{F}_s}{E_pE_qE_kE_{k'}}, \quad (43)$$

where

$$\mathcal{F}_s = (p\cdot k')\,(q\cdot k) \mp m_\mu (e\cdot k')\,(q\cdot k) = \mathcal{F}\mp\mathcal{F}_e, \quad (44)$$

where $\mathcal{F}_e$ is the decay amplitude related to the polarization vector e:

$$\mathcal{F}_e = m_\mu (e\cdot k')\,(q\cdot k). \quad (45)$$

Obviously, decay amplitude $\mathcal{F}_s$ is also a Lorentz scalar, like $\mathcal{F}$. Therefore we have

$$\mathcal{F}_s = \mathcal{F}_s^0 = (p^0\cdot k')\,(q\cdot k) \mp m_\mu (e^0\cdot k')\,(q\cdot k), \quad (46)$$

where $\mathcal{F}_s^0$ is $\mathcal{F}_s$ in the muon rest frame.

In a similar way to Eq. (36), we obtain

$$\begin{aligned}\tau_s^{-1} &= \frac{4\,G^2}{(2\pi)^5}\frac{1}{E_p}\int\frac{d^3q}{E_q}\frac{d^3k}{E_k}\frac{d^3k'}{E_{k'}}\delta^4(p-q-k-k')\mathcal{F}_s \\ &= \tau^{-1}+\tau_e^{-1}. \quad (47)\end{aligned}$$

One easily verifies that the second integration over $\mathcal{F}_e$ vanishes, i.e., $\tau_e = 0$. The first integration is identical with Eq. (36) and we obtain

$$\tau_s = \tau. \quad (48)$$

It is also easy to see in the above calculation that the lifetime in the laboratory frame does not exhibit any lifetime asymmetry when the polarization of muons is described by spin states.

C. The lifetime of polarized muons expressed by the helicity states

As mentioned above, however, this method does not enable us to discuss the dependence of lifetime on the polarization of muons. Because a spin state is helicity degenerate, the polarization of muons must be described by helicity states. For LH polarized muons, substituting the spin states in Eq. (41) with the helicity states and considering Eq. (16), we obtain

$$\begin{aligned}I_{Lh} &= \sum_h \left[\overline{u}_h(k)\gamma_\lambda(1+\gamma_5)u_{Lh}(p)\right]^2 \\ &= \sum_{r=1}^{2}\left[\overline{u}_r(k)\gamma_\lambda(1+\gamma_5)u_{Lh}(p)\right]^2. \quad (49)\end{aligned}$$

From Eqs. (25), (28) and (22) we easily find

$$(1+\gamma_5)u_{Lh}(p) = 2\sqrt{1+\beta}\,u_{L2}^0 = \sqrt{1+\beta}(1+\gamma_5)u_2^0, \quad (50)$$

where u_2^0 is the spin state in the muon rest frame. One can see that the chirality-state projection operator $(1+\gamma_5)$ picks out LH chirality state u_{L2}^0 in a LH helicity state, which is factorized into two parts in the second equation, one is the spin state u_2^0 and another is a factor $\sqrt{1+\beta}$ which depends on muon's helicity. Substituting Eq. (50) into Eq. (49) we have

$$I_{Lh} = (1+\beta)\sum_{r=1}^{2}\left[\overline{u}_r(k)\gamma_\lambda(1+\gamma_5)u_2^0\right]^2 \quad (51)$$

Comparing Eq. (51) with Eq. (41) and considering Eq. (46) we find out the decay amplitude of LH polarized muons

$$\mathcal{F}_{Lh} = (1+\beta)\mathcal{F}_2^0 = (1+\beta)\mathcal{F}_2. \quad (52)$$

Then the LH polarized fermion lifetime is given by

$$\tau_{Lh}^{-1} = \frac{4\,G^2}{(2\pi)^5}\frac{1}{E_p}\int\frac{d^3q}{E_q}\frac{d^3k}{E_k}\frac{d^3k'}{E_{k'}}\delta^4(p-q-k-k')\mathcal{F}_{Lh}. \quad (53)$$

Similarly, for RH polarized muons we obtain

$$\begin{aligned}I_{Rh} &= \sum_h \left[\overline{u}_h(k)\gamma_\lambda(1+\gamma_5)u_{Rh}(p)\right]^2 \\ &= (1-\beta)\sum_{r=1}^{2}\left[\overline{u}_r(k)\gamma_\lambda(1+\gamma_5)u_1^0\right]^2 \quad (54)\end{aligned}$$

and the decay amplitude is

$$\mathcal{F}_{Rh} = (1-\beta)\mathcal{F}_1^0 = (1-\beta)\mathcal{F}_1. \quad (55)$$

Then the RH polarized fermion lifetime is given by

$$\tau_{Rh}^{-1} = \frac{4\,G^2}{(2\pi)^5}\frac{1}{E_p}\int\frac{d^3q}{E_q}\frac{d^3k}{E_k}\frac{d^3k'}{E_{k'}}\delta^4(p-q-k-k')\mathcal{F}_{Rh}. \quad (56)$$

Comparing Eqs. (52) and (55) with Eq. (44), respectively and considering Eqs. (47), (48) and (40), we find the polarized muon lifetime

$$\tau_{Lh} = \frac{\tau}{1+\beta}, \quad \text{and} \quad \tau_{Rh} = \frac{\tau}{1-\beta}. \quad (57)$$

It is easy to see that the τ_{Rh} is greater than τ_{Lh}, which shows the lifetime asymmetry of left-right handed polarized fermions. The lifetime asymmetry is expressed by

$$\text{lifetime asymmetry} \equiv \frac{\tau_{Rh}-\tau_{Lh}}{\tau_{Rh}+\tau_{Lh}} = \beta. \quad (58)$$

When $\beta = 0$, we find $\tau_{Rh} = \tau_{Lh} = \tau_0$; when $\beta \to 1$, we find $\tau_{Rh} \to \infty$, $\tau_{Lh} \to \infty$. In particular, when $\beta = \frac{1}{2}$, the lifetime of the LH polarized fermions has a minimum value $\tau_{Lh} = \tau_{min} = 0.77\tau_0$, as shown in Fig. 1.

It is not difficult to prove that the lifetimes of antimuons μ^+ in flight are given by

$$\overline{\tau}_{Rh} = \frac{\tau}{1+\beta}, \quad \text{and} \quad \overline{\tau}_{Lh} = \frac{\tau}{1-\beta}. \quad (59)$$

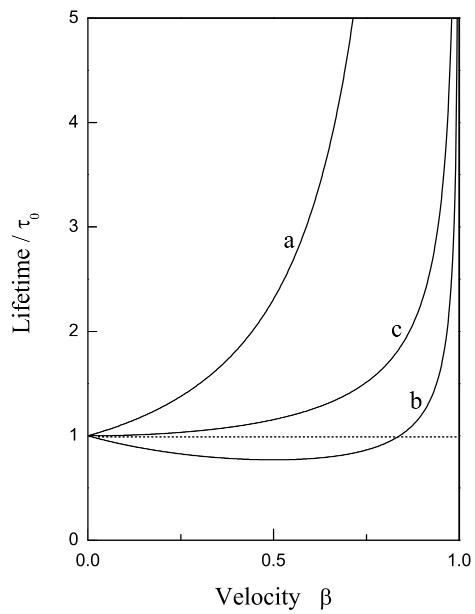

FIG. 1: Lifetime as a function of muon velocity β. (a) The lifetime τ_{Rh} of right-handed polarized muons. (b) The lifetime τ_{Lh} of left-handed polarized muons. (c) The lifetime τ of unpolarized muons.

V. SUMMARY AND DISCUSSION

The calculation has established that the lifetime of RH polarized muons is different from that of LH polarized muons in flight. Under a space reflection the LH helicity state Eq. (25) and the RH helicity state Eq. (26) transform to each other, therefore the set of equations Eq. (57) and Eq. (59) are valid all the same, respectively. Furthermore, this conclusion is also valid for all fermions in the decays under weak interactions. It means that the τ_{Lh} is always smaller than τ_{Rh} for fermions and the $\overline{\tau}_{Rh}$ smaller than $\overline{\tau}_{Lh}$ for antifermions in any one of inertial systems in which fermions or antifermions are in flight with a same speed. The lifetime asymmetry shows a maximum violation of parity symmetry. Hence under the condition of parity violation the lifetime is neither a four-dimensional scalar, nor a scalar under the three-dimensional space inversion.

We emphasize here an important concept that a spin state is helicity degenerate and the spin projection operators $\rho_{\pm}$ can only project out the spin states, but can not project out the helicity states. The above calculation shows that the so-called eigenstate of operator given by Eq. (8) is by no means a helicity eigenstate (even we had chosen $\boldsymbol{p}$ vector along z axis) and this is why the parity violation result, Eqs. (57) and (59), was overlooked in the past for so long a time even one did not perform the spin average for muons in the laboratory frame. Therefore, the polarized fermions must be expressed by the helicity states which are relevant to physical interpretation and experimental test.

If we find out nonzero pseudoscalar of similar $\langle \boldsymbol{\sigma} \cdot \boldsymbol{p} \rangle$ in a physical process, then parity conservation law is certainly violated and Lorentz invariance is also violated because $\langle \boldsymbol{\sigma} \cdot \boldsymbol{p} \rangle$ is not a Lorentz scalar. In the special theory of relativity space coordinates are treated on an equal footing with time coordinate. The violation of pure space inversion, in some sense, implies a violation to the equal status and mutual transformation between space and time. And it in turn implies that something might go beyond the theory of special relativity[14]. The lifetime asymmetry is just such an example. It poses a serious challenge to beautiful symmetry in physics.

When the polarization of muons is described by spin states, it can be seen from Eq. (45) that the factor related to polarization reads

$$(e \cdot k') = (e \cdot p) - (e \cdot q) - (e \cdot k). \tag{60}$$

The pseudoscalars $\boldsymbol{e} \cdot \boldsymbol{q}$ and $\boldsymbol{e} \cdot \boldsymbol{k}$ are included in it, which reveals the asymmetric distribution of electrons and neutrinos emitted from decaying muons and exhibits parity nonconservation in the final channels. The property $(e \cdot p) = 0$ ensures the pseudoscalar $\boldsymbol{e} \cdot \boldsymbol{p}$, helicity operator, does not appear in the decay amplitude. Therefore, the decay amplitude corresponding to the spin states is independent of muon helicity, which is consistent with the helicity degeneracy of the spin states.

Differing from the spin states, however, for massive muons LH and RH helicity state may transform to each other as observer's velocity paralleling and exceeding the muon's one. If, therefore, the decay amplitude corresponding to the helicity state were a Lorentz scalar, then the decay amplitude of LH polarized muons would be equal to that of RH one and starting from Eqs. (52), (55) and (44), we would obtain $\mathcal{F}_{Lh} = \mathcal{F}_{Rh} = (p \cdot k')(q \cdot k)$. Obviously, there exists no any pseudoscalars in it, which shows that the parity is conserved and is clearly wrong. Hence we conclude that the decay amplitude $\mathcal{F}_{Lh}$ and $\mathcal{F}_{Rh}$ corresponding to the helicity states must be not Lorentz invariant, as shown Eqs. (52) and (55). It is the root cause of the lifetime asymmetry. In other words, if the polarization is described by the helicity states, then the lifetime asymmetry will be inevitable. Of course, the average value of decay amplitude, $\frac{1}{2}(\mathcal{F}_{Lh} + \mathcal{F}_{Rh})$, is still a Lorentz scalar.

In the before, physicists too believed in the covariant form of four spinor which enjoys the invariance under the LT and so misunderstood the meaning of so-called spin states . Meanwhile, the importance of helicity state and its difference from spin state and chiral state were often confused or overlooked. Our lesson and experience are just focused on the above crucial point before we are able to get rid of the constraint of covariant four spinor form. What we eventually realized is to discriminate two things: while a physical law like that reflected by the Dirac equation is one thing, a phenomenon like a particle's velocity, its helicity as well as its decay lifetime is

another thing. A law should be invariant under the LT , but a phenomenon needs not. Actually, a phenomenon may be different in different inertial systems. In short, the information is not existing totally in the sense of objectiveness, it is created by the subject and object in common.

The measurements on muon decay used to be performed in its rest frame. It was realized that muons, formed by forward decay in flight of pions inside cyclotron, were stopped in a nuclear emulsion, sulphur, carbon, calcium or polyethylene target. The polarization effects of muon decay were observed using carbon stopping target[15], in which there is no depolarization of the muons. So far the measurement of the lifetime of polarized muons in flight has not yet been found in literature. Therefore, one actually lacks any direct experimental evidence either to support or to refute the lifetime asymmetry. We report it here now in the hope that it may stimulate and encourage further experimental investigations on the question of the lifetime asymmetry in muon decays.

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